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Viscous Dissipation and Joule Heating Effect on Unsteady MHD Free Convection Fluid Flow along the Vertical Porous Plate in Presence of Ion-Slip Current

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Abstract
The viscous dissipation and Joule heating effect on unsteady MHD free convection fluid flow along a semi infinite vertical porous plate in presence of ion-slip current has investigated. The governing equations have been transformed into dimensionless coupled nonlinear ordinary differential equations by applying usual transformations. The finite difference method is used as a main tool for the numerical solution. The obtained numerical results are presented in the form of primary and secondary velocities; and temperature distributions for different parameters entering into the problem. The results show that the primary velocity increases for different values of ion-slip parameter, Eckert number while it decreases for different values of magnetic parameter and Prandtl number. Temperature distributions increase with the increase of magnetic parameter and Eckert number while it decreases with the increases of Prandtl number. The results of velocity and temperature distributions are displayed in the form of graph and also the local skin friction coefficient and Nusselt number are also shown in the form of tabular.

Keywords: MHD, Ion-slip current, viscous dissipation, Joule heating, uniform magnetic field.

1. Introduction
The natural convection flow on a vertical surface embedded in porous media occurs in many important engineering problems such as heat exchanger devices, petroleum reservoirs, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal. Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Prasanna Lakshmi et al. [1] studied MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction and viscous dissipation. Khaled [2] studied the influence of Hall current and viscous dissipation on MHD convective heat and mass transfer in a rotating porous channel with joule heating. In an ionized gas where the density is low and/or the magnetic field is very strong, the effects of Hall and ion-slip currents play a significant role in the velocity distribution of the flow. The study of magnetohydrodynamic flows with Hall and ion-slip currents has important engineering applications in the problem of magnetohydrodynamic generators and of Hall accelerators as well as flight magnetohydrodynamics. Emad M. Abo-Eldahab and ,Mohamed A. El Aziz [3] studied viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. Combined effect of viscous dissipation and joule heating on the coupling of conduction and free convection along a vertical flat plate investigated by Alim et al [4]. Mamun et al. [5] investigated combined effect of conduction and viscous dissipation on MHD free convection flow along a vertical flat plate. Md. Mahmud Alam et al. [6] studied viscous dissipation and joule heating effects on steady MHD combined heat and mass transfer flow through a porous medium in a rotating system. Combined effects of Hall and ion-slip currents on free convective heat generating flow past a semi-infinite vertical flat plate have been investigated by Abo-Eldahab and Aziz [7]. Koushik Dash et al. [8] studied MHD free convection and mass transfer flow from

a vertical plate in the presence of Hall and ion-slip current. Ferdows et al.[9] investigated the effects of Hall and ion-slip currents on free convective heat transfer flow past a vertical plate considering slip conditions. Rama Krishna Reddy and Raju [10] studied MHD free convective flow past a porous plate. Anjali Devi et al. [11] investigated the Hall effect on unsteady MHD free convection flow past an impulsively started porous plate with viscous and Joule's dissipation. Joule heating effect on Magnetohydrodynamic natural convection flow along a vertical wavy surface studied by Nazma Parveen and Alim [12].

Hence, our objective is to investigate viscous dissipation and Joule heating effect on unsteady MHD free convection fluid flow along the vertical semi-infinite porous plate in presence of ion-slip current.

2. Governing Equations

The two dimensional unsteady flow of an electrically conducting incompressible viscous fluid past an semi-infinite vertical porous plate has been considered. The flow is assumed to be in the x-axis which is taken along the plate in the upward direction and y-axis is normal to it. Initially the fluids as well as the plate are at rest. It is assumed that, T_w are temperature and spices concentration at the wall and, T_∞ are the temperature and the concentration of the spices outside the boundary layer respectively. The physical configuration of the problem is shown in Fig.1. A strong magnetic field is applied in the y-direction. The uniform magnetic field strength B_0 can be taken as $\mathbf{B} = (0, B_0, 0)$. The induced magnetic field is neglected, since the magnetic Reynolds number of a partially-ionized fluid is very small. The equation of conservation of electric charge $\overline{\nabla} \bullet \mathbf{J} = 0$ gives $J_y = \text{constant}$ because the direction of propagation is considered only along y-axis and $\mathbf{J}$ does not have any variation along the y-axis. The equations which govern the flow under the above consideration and Boussinesq's approximation are as **fo**llows:

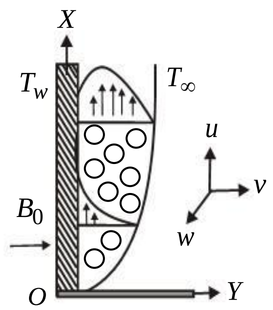


Fig.1 Physical configuration and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g_0\beta(T - T_\infty) - \frac{\upsilon}{k}u - \frac{\sigma_e B_0^2}{\rho(\alpha_e^2 + \beta_e^2)}(\alpha_e u + \beta_e w) \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \upsilon\frac{\partial^2 w}{\partial y^2} - \frac{\upsilon}{k}w + \frac{\sigma_e B_0^2}{\rho(\alpha_e^2 + \beta_e^2)}(\beta_e u - \alpha_e w) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma_e B_0^2}{\rho c_p}(u^2 + w^2) \tag{4}$$

where $\alpha_e = 1 + \beta_e\beta_i$, β_e (Hall parameter), β_i (ion-slip parameter), β (volumetric coefficient of thermal expansion), g_0 (acceleration due to gravity), υ (Kinematic viscosity), ρ (fluid density), k (permeability of the porous medium), c_p (Specific heat at constant pressure), κ (Thermal conductivity), B_0 (uniform magnetic field), σ_e (Electrical conductivity), T (temperature in the boundar**y** layer), T_∞ (temperature outside the boundary layer), t (dimensional time).

The boundary conditions for the problems are;

$$\begin{aligned} &u = U_0, v = 0, w = 0, T = T_w, \quad \text{at } y = 0 \\ &u = 0, v = 0, w = 0, T = T_\infty, \quad \text{as } y \to \infty \end{aligned} \tag{5}$$

3. Mathematical Formulation

The problem is simplified by writing the equations in the non-dimensional form. Now introduce the following non-dimensional quantities

$$X = \frac{xU_0}{\upsilon},\ Y = \frac{yU_0}{\upsilon},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ W = \frac{w}{U_0},\ \tau = \frac{tU_0^2}{\upsilon},\ \overline{T} = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Then introducing the dimensionless quantities (6) in equations (1)-(4) respectively, the following dimensionless equations are as follows;

$$\frac{\partial U}{\partial X}+\frac{\partial v}{\partial X}=0 \tag{7}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+G_r\overline{T}-\gamma U-\frac{M\left(\alpha_e U+\beta_e W\right)}{\alpha_e^2+\beta_e^2} \tag{8}$$

$$\frac{\partial W}{\partial \tau}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\frac{\partial^2 W}{\partial Y^2}-\gamma W+\frac{M\left(\beta_e U-\alpha_e W\right)}{\alpha_e^2+\beta_e^2} \tag{9}$$

$$\frac{\partial \overline{T}}{\partial \tau}+U\frac{\partial \overline{T}}{\partial X}+V\frac{\partial \overline{T}}{\partial Y}=\frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial Y^2}+E_c\left[\left(\frac{\partial U}{\partial Y}\right)^2+\left(\frac{\partial W}{\partial Y}\right)^2\right]+E_cM\left[U^2+W^2\right] \tag{10}$$

The corresponding boundary conditions are as follows;

$$U=1, V=0, W=0, \overline{T}=1, \quad at\ Y=0$$

$$U=0, \quad W=0, \overline{T}=0, \quad as\ Y\to 0 \tag{11}$$

where $G_r=\frac{g_0\beta(T_w-T_\infty)\upsilon}{U_0^3}$ (Grashof number), $M=\frac{\sigma_e B_0^2\upsilon}{\rho U_0^2}$ (magnetic parameter), $P_r=\frac{\rho\upsilon c_p}{\kappa}$ (Prandtl number), $\gamma=\frac{\upsilon^2}{kU_0^2}$ (permeability parameter), $E_c=\frac{U_0^2}{c_p\left(T_w-T_\infty\right)}$ (Eckert number).

4. Solution Technique

The governing second order non-linear coupled dimensionless partial differential equations have been solved numerically with the associated boundary conditions by Compaq visual Fortran 6.6a and the figures have been drawn by Tecplot 7. The explicit finite difference method has been used to solve the coupled equations (7)-(10) with boundary conditions (11).To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes, where X-axis is taken along the plate and Y-axis is taken normal to the plate. Here the plate height $X_{\max}(80.0)$ is considered i.e. X varies form 0 to 80 and assumed $Y_{\max}(60.0)$ as corresponding $Y_{\max}\to\infty$ i.e. Y varies from 0 to 60. There are $m=300$ and $n=300$ grid spacing in the X and Y directions respectively and taken as follows $\Delta X=0.27(0\le X\le 80)$
and $\Delta Y=0.2(0\le Y\le 60)$ with the smaller time step $\Delta\tau=0.005$.

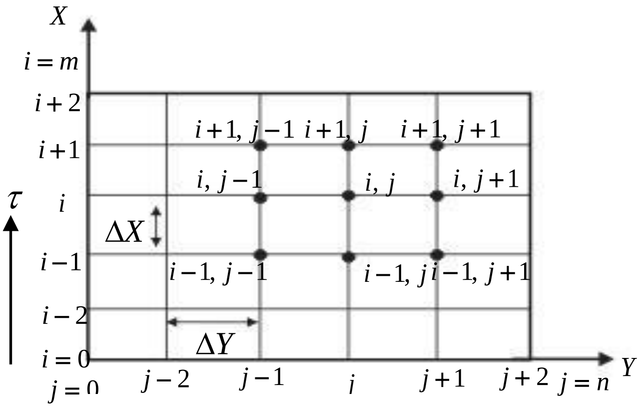


Fig.2 Finite difference grid space

5. Shear Stress, Nusselt number and Sherwood number

The quantities of chief physical interest are shear stress, Nusselt number and Sherwood number. The following equations represent the local and average shear stress at the plate. Local shear stress in x and z-axes are as follows; $\tau_{LU}=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $\tau_{LW}=\mu\left(\frac{\partial w}{\partial y}\right)_{y=0}$ which are proportional to $\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ and $\left(\frac{\partial W}{\partial Y}\right)_{Y=0}$. The following equations define the x and z components of the average shear stress $\tau_{AU}=\mu\int_0^{80}\left(\frac{\partial u}{\partial y}\right)_{y=0}dx$ and

$\tau_{AW} = \mu \int_0^{80} \left(\frac{\partial w}{\partial y} \right)_{y=0} dx$ which are proportional to $\int_0^{80} \left(\frac{\partial U}{\partial Y} \right)_{Y=0} dX$ and $\int_0^{80} \left(\frac{\partial W}{\partial Y} \right)_{Y=0} dX$. The local and average Nusselt numbers are denoted by N_{uL}, N_{uA} which are proportional to $-\left(\frac{\partial \overline{T}}{\partial Y} \right)_{Y=0}$ and $-\int_0^{80} \left(\frac{\partial \overline{T}}{\partial Y} \right)_{Y=0} dX$.

6. Results and Discussion

The numerical results has been carried out for dimensionless primary velocity (U), secondary velocity(W), temperature($\overline{T}$), local and average shear stresses in x-axis (τ_{LU}, τ_{AU}), local and average shear stresses in z-axis (τ_{LW}, τ_{AW}), local and average Nusselt numbers (N_{uL}, N_{uA}) for various values of the material parameters such as Hall parameter(β_e), ion-slip parameter(β_i), magnetic parameter(M), Prandtl number(P_r), permeability parameter(γ), Eckert number (E_c). The values for the parameters are chosen arbitrarily in most cases. Some standard values for of the Prandtl number (P_r) is considered because of the physical importance. Physically $P_r = 0.71$ corresponds to air at $20^0 C$, $P_r = 1.0$ corresponds to water at $20^0 C$, $P_r = 1.63$ corresponds to glycerin at 50^0C. The importance of cooling problem in nuclear engineering in connection with the cooling of reactors, the values of G_r is taken positive. Throughout the calculations the values of G_r is taken very large ($G_r = 5.0$). From Fig.3 (a-c), it have been seen that the primary velocity (U), local and average shear stresses in x-axis (τ_{LU}, τ_{AU}) decrease with the increase of magnetic parameter (M). An increase in the value of the magnetic parameter (M) leads to increase in the magnitude of the Lorentz force which serves to retard the primary velocity. Fig.4 (a-c) is illustrated that the temperature $\overline{T}$ distributions increase whereas local and average Nusselt (N_{uL}, N_{uA}) decrease with the increase of M. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to increase its temperature distributions. Analyzing the Fig.5 (a) it is clearly seen that the primary velocity (U) profiles increases with an increase Eckert number (E_c). This is due to the heat energy stored in the liquid because of the frictional heating. The local and average shear stresses (τ_{LU}, τ_{AU}) in x-axis increase for increasing values of Eckert number which are shown in Fig.5 (b,c). Fig.6 (a-c) is illustrated that the temperature $\overline{T}$ distributions increase whereas local and average Nusselt (N_{uL}, N_{uA}) decrease with the increase of E_c. The increase in the buoyancy force due to an increase in the Eckert number enhances the temperature. Numerical values of the local shear stress in x and z- axes, Nusselt number for Prandtl's number and permeability parameter are shown in Table 1. Qualitative comparison of the present results with the previous results in tabular form is shown in Table 2. The accuracy of the present results is qualitatively good in case of all the respective flow parameters. Other results are not shown for brevity.

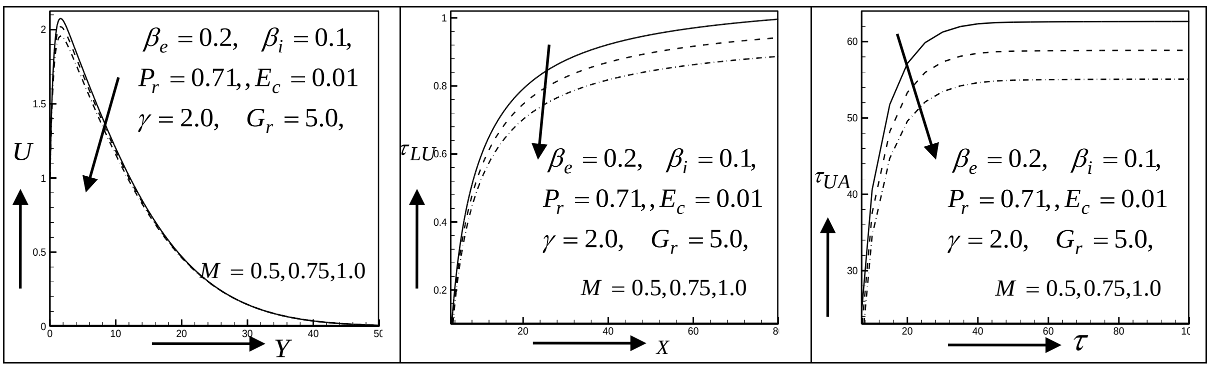
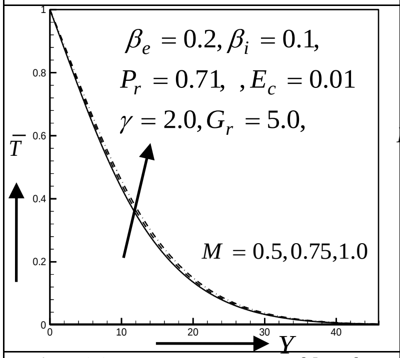
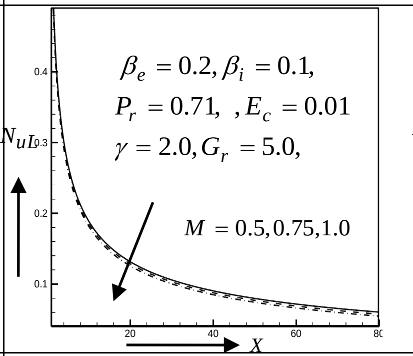


Fig.3(a) Primary velocity profiles for different values of M

Fig.3(b) Local Shear stress in x-axis for different values of M

Fig.3(c) Average Shear stress in x-axis for different values of M





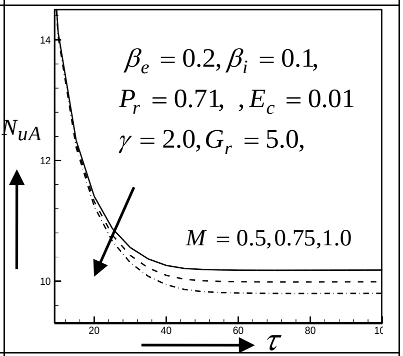
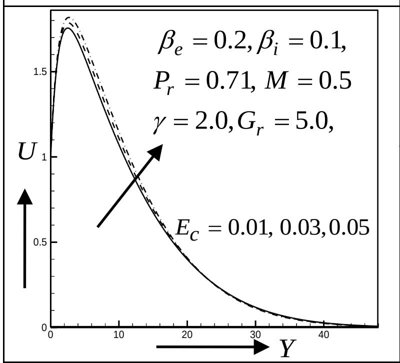
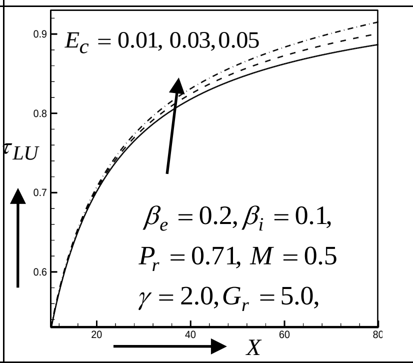


Fig.4(a) Temperature profiles for different values of M

Fig.4(b) Local Nusselt number for different values of M

Fig.4(c) Average Nusselt number for different values of M





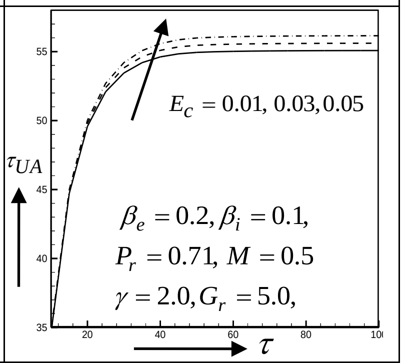
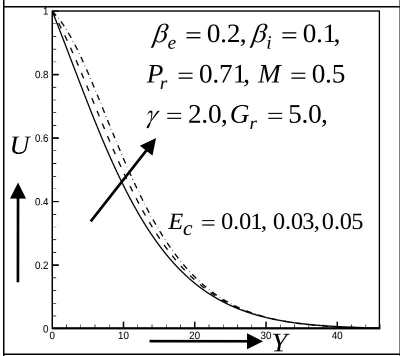
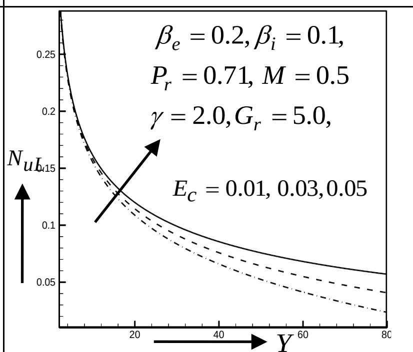


Fig.5(a) Primary velocity profiles for different values of E_c

Fig.5(b) Local Shear stress in x-axis for different values of E_c

Fig.5(c) Average Shear stress in x-axis for different values of E_c





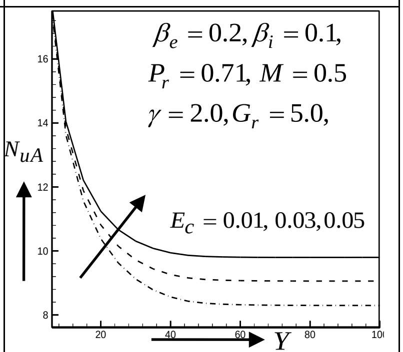


Fig.6(a) Temperature profiles for different values of E_c

Fig.6(b) Local Nusselt number for different values of E_c

Fig.6(c) Average Nusselt number for different values of E_c

Table 1. Numerical values of τ_{LU}, τ_{LW}, τ_z and N_{uL} for $G_r = 5.0,\ \beta_e = 0.2,\ \beta_i = 0.1,\ E_c = 0.01,\ M = 0.5$

G_r	P_r	γ	τ_{LU}	τ_{LW}	N_{uL}
+ve	0.71	2.0	0.8523298	1.4003298	0.7103118
	1.0	2.0	0.6584448	1.5584448	0.9184008
	1.63	2.0	0.4877677	1.8877677	1.0077107
	0.71	3.0	0.7270881	1.5270881	0.8062081
	0.71	4.0	0.5532185	1.6832185	1.0512105

Table 2. Qualitative comparison of the present results with the previous results.

Increased parameters	Previous results given by Abo-Eldahab and El Aziz [3]		Present results	
	f'	θ	U	$\bar{T}$
M	Dec	Inc	Dec	Inc
E_c	Inc	Inc	Inc	Inc

7. Conclusion

From above mentioned studies, following conclusion can be drawn:

(i) Magnetic field has significant effect on primary velocity field and retards the motion of the fluid.

(ii) Temperature distribution increase with the increase of magnetic parameter and Eckert number.

(iii) Primary velocity profiles increase with increase of Eckert number.

(iv) The local and average shear stress in x-axis, local and average Nusselt number are decreased with an increase of magnetic parameter and Eckert number.

(v) The shear stress in x-axis increases with an increase of Eckert number.

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